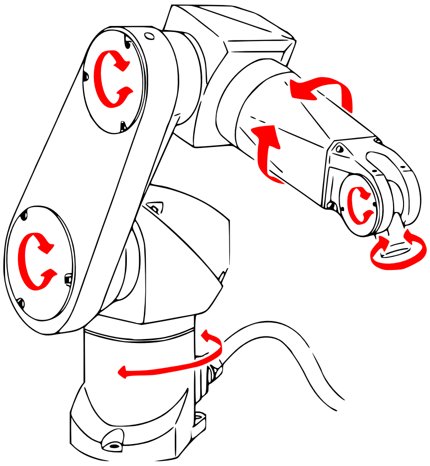
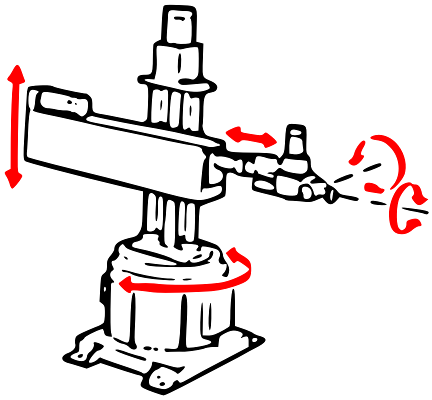
Exercise set 1 – Geometry – Solutions

Exercise 1

For the following two structures:



(a) Stäubli TX60



b)

What is the:

1. Number of motors?
2. Mobility (MO)?
3. Number of degrees of freedom (DOF)?

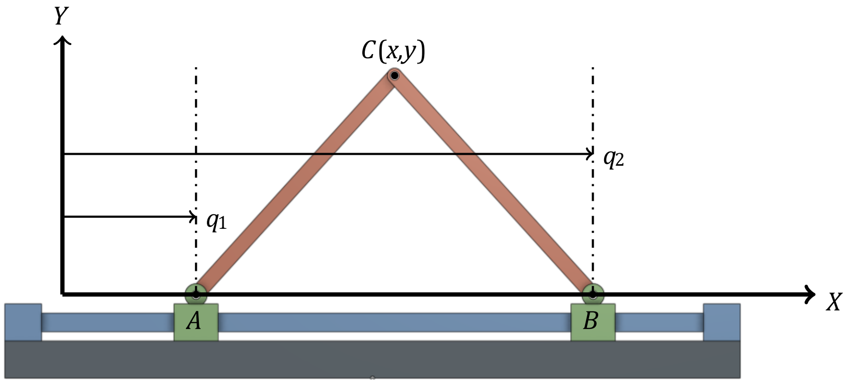
Solution 1

Reminder : The mobility of a serial robot is always equal to its number of motors. All the joints of a serial robot are actuated (motorized).

1. (a) The Stäubli TX60 has 6 motorized joints: RRR for the handler and RRR for the wrist.
   (b) The second robot has 5 motorized joints: RTT for the handler and RR for the wrist.
2. (a) Stäubli TX60: MO = 6.
   (b) Second robot: MO = 5.
3. (a) Stäubli TX60: DOF = 6; three translations of the tool and three rotations in the space of the tool.
   (b) Second robot: DOF = 5; three translations of the tool and two rotations in the space of the tool.

Exercise 2

Consider the following Lambda robot:

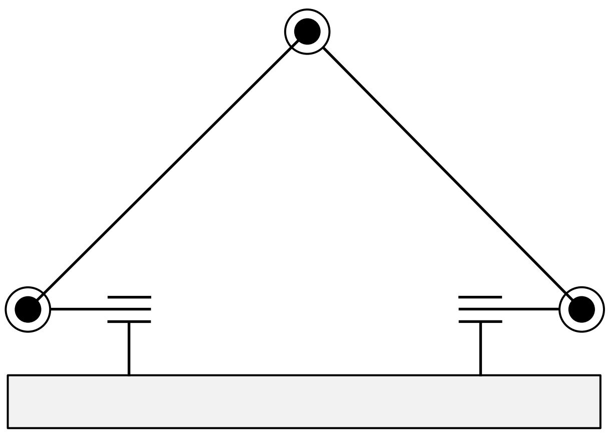


Each rotational joint is a pivot.

1. **Give the kinematic representation of the structure.**
2. **Calculate the mobility of this structure:**
   (a) by Grübler's formula.
   (b) by the formula of loops.
3. **Give the number of degrees of freedom.**
4. **By comparing the DOF (degrees of freedom) and the MO (mobility), discuss if the structure is over-constrained (fr: hyperguidée) or not.**

Solution 2

1. Kinematic representation of the structure:



2. We have n = 5 the number of parts (base included), k = 5 the number of joints and bo = 1 the number of closed kinematic loops. The MO_i are the mobilities associated with each articulation, for i ranging from 1 to k.

$$\sum_{i=1}^{k} \mathrm{MO}_i = 5 \cdot 1 = 5.$$

   (a) Grübler's formula:

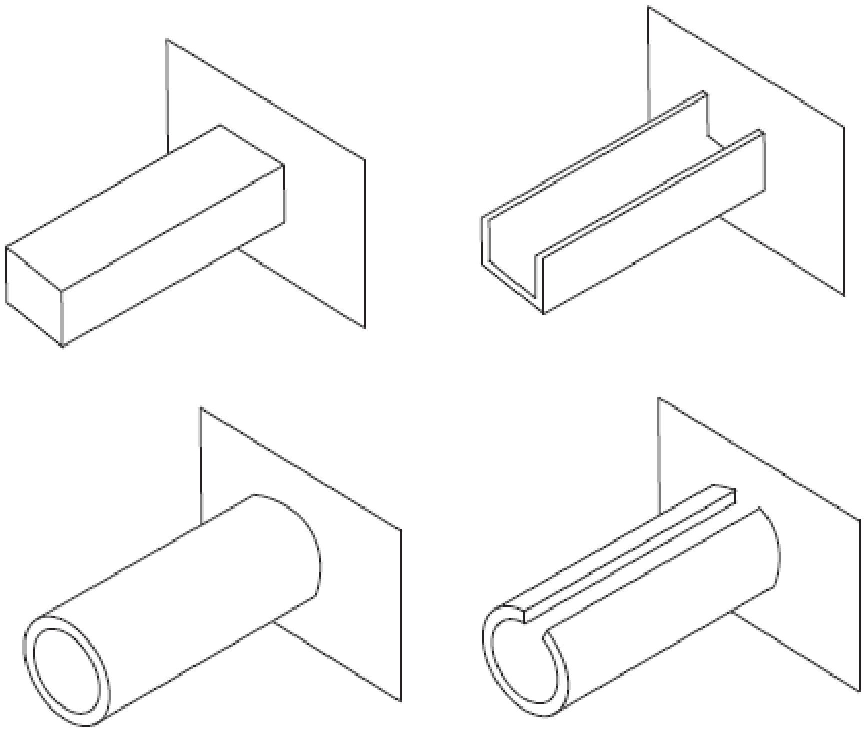
$$\begin{aligned}\mathrm{MO} &= 6(n-k-1)+\sum_{i=1}^{k}\mathrm{MO}_i\\ &= 6(5-5-1)+5\\ &= -1\end{aligned}$$

(b) Loops formula:

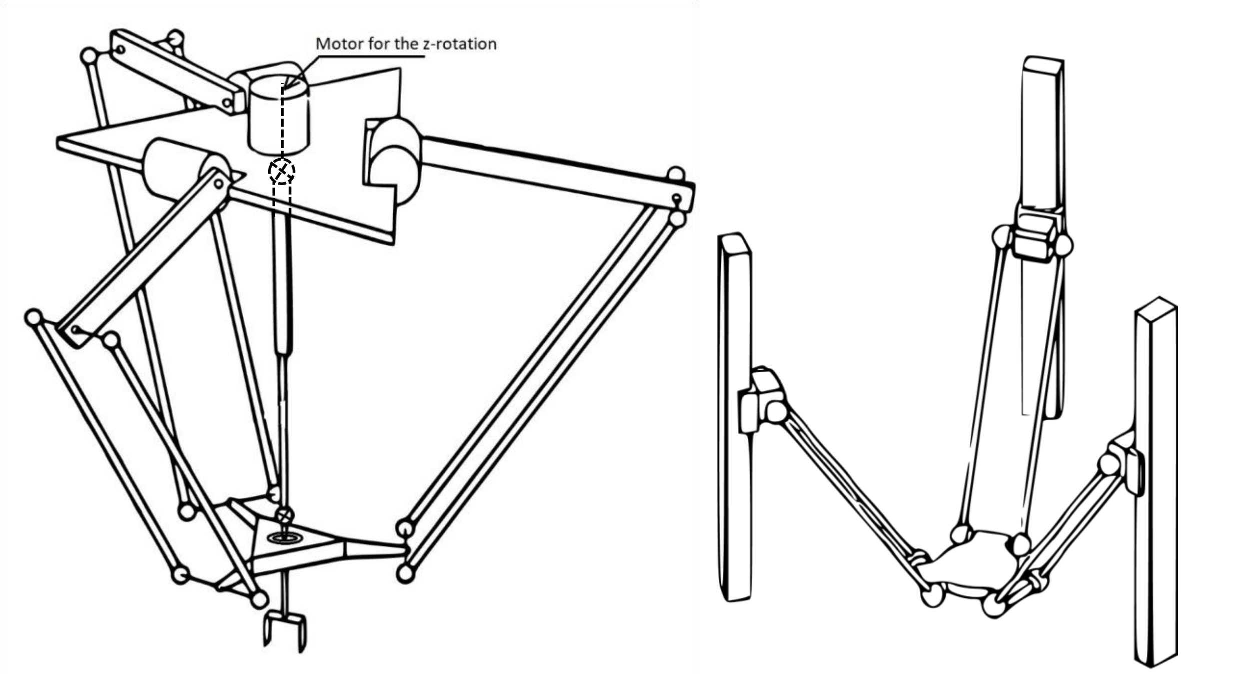
$$\begin{aligned}\mathrm{MO} &= \sum_{i=1}^{k}\mathrm{MO}_i - 6\mathrm{bo}\\ &= 5-6\\ &= -1\end{aligned}$$

This structure has 2 DOF which correspond to the displacements of the point C in the directions X and Y.

3. It is over-constrained (fr: hyperguidée) because MO = −1 < 2 = DOF. The reason is that there are two assembly planes of the structure. The first plane is defined by the rotation of the arm AC about pivot A and the second plane is defined by the rotation of the arm BC around pin B. To remove this over-constraint (fr: hyperguidage), it would be necessary to add 3 mobilities and thus obtain MO = DOF = 2. One could for example put a universal joint at each pivot type articulation or add torsional weaknesses at the level of the arms (for example, changing the cross-sectional area of a component by using U-shaped profiles instead of rectangular profiles or by removing a section from a circular cross-sectional area.).



Exercise 3



a) Delta 4 (4 DOF delta robot) b) Linear delta (the three vertical axes are part of the same base)

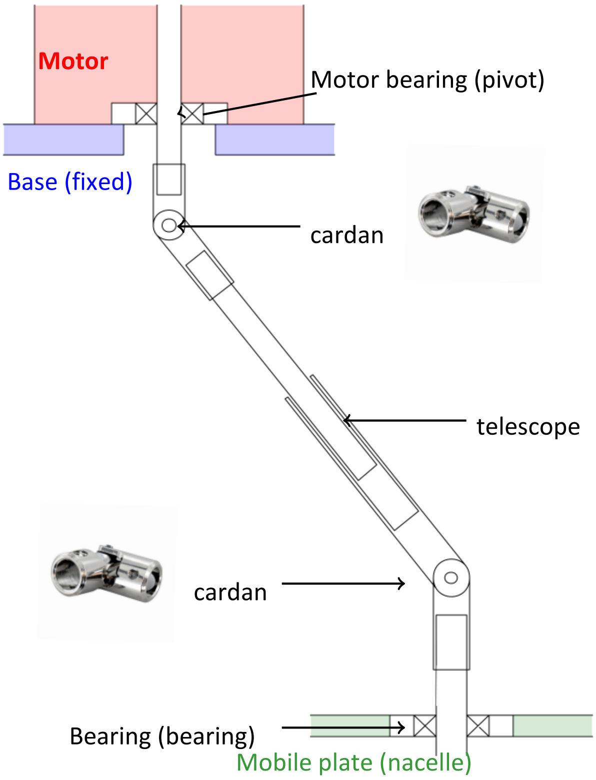
For each structure:

1. Give the kinematic representation.

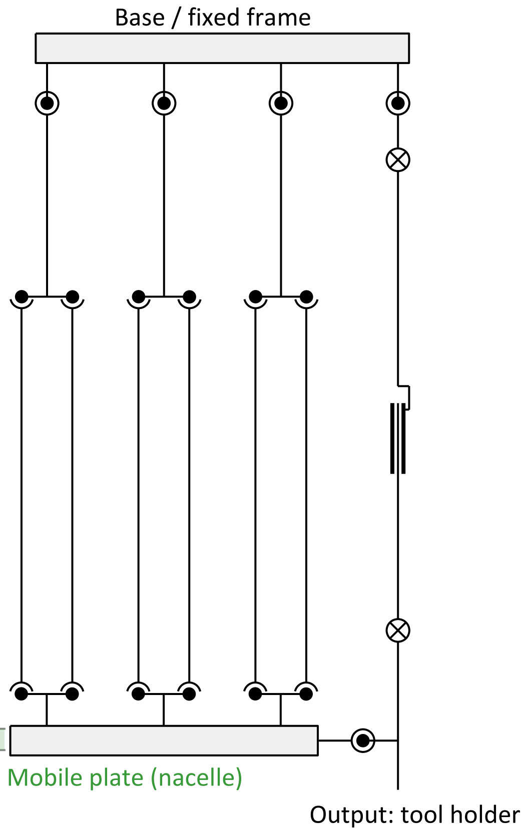
   For the Delta 4, think about the functional diagram of the telescopic arm that helps in the construction of the entire kinematic representation.
2. Calculate the mobility:
   (a) by Grübler's formula.
   (b) by the formula of loops.

Solution 3

Kinematic representations:



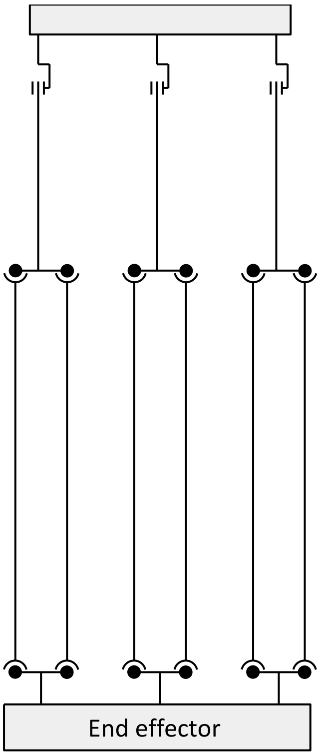
Functional / mechanical illustration of the robot Delta 4



Kinematic representation of the Delta 4

The Delta 4 robot has 4 DOFs: three translations of the mobile plate and one rotation of the tool holder (see the figure above). This tool holder can rotate around a pivot (bearing) located on the mobile plate.

The motor on the base (in red) is responsible for the 4th DOF; the rotation is transmitted to the tool via a telescopic arm and two gimbals, one at each end of the telescopic arm.



Kinematic representation of the linear Delta

2. We start by counting, for each robot, the number of parts, the number of joints, the number of closed loops, and the sum of the mobilities associated with the joints:

For the Delta 4, we have:

Number of elements:

n_1 = 1+(3*3)+1+ 4 = 15 (base, 3x{1 arm + 2 forearms (parallel bars)}, mobile plate, telescopic arm (4 parts))

Number of joints:

k_1 = 3*5 + 5 = 20

Number of loops:

$\mathrm{bo}_1 = 6$

Sum of mobilities:

$$\sum_{i=1}^{k_1} \mathrm{MO}_{1,i} = (1 + 4 \cdot 3) \cdot 3 + (1 + 2 + 1 + 2 + 1) = 39 + 7 = 46$$

(a) Grübler's: formula

$$\begin{aligned} \mathrm{MO}_1 &= 6(n_1 - k_1 - 1) + \sum_{i=1}^{k_1} \mathrm{MO}_{1,i} \\ &= 6(15 - 20 - 1) + 46 \\ &= 10 \end{aligned}$$

(b) Loops formula:

$$\begin{aligned} \mathrm{MO}_1 &= \sum_{i=1}^{k_1} \mathrm{MO}_{1,i} - 6\mathrm{bo}_1 \\ &= 46 - 6 \cdot 6 \\ &= 10 \end{aligned}$$

For the linear Delta, we have:

Number of elements :

n_2 = 1 + (3*3) + 1 = 11 (base, 3x (1 linear carriage + 2 parallel bars), mobile plate)

Number of joints :

k_2 = 3*5 = 15

Number of loops:

$\mathrm{bo}_2 = 5$

Sum of mobilities:

$$\sum_{i=1}^{k_2} \mathrm{MO}_{2,i} = (1 + 4 \cdot 3) \cdot 3 = 39$$

(a) Grübler's: formula

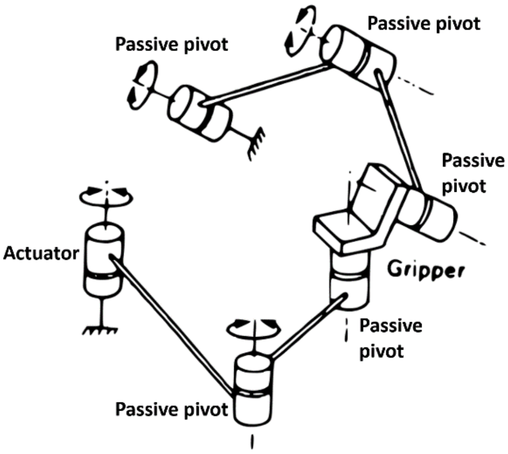
$$\begin{aligned} \mathrm{MO}_2 &= 6(n_2 - k_2 - 1) + \sum_{i=1}^{k_2} \mathrm{MO}_{2,i} \\ &= 6(11 - 15 - 1) + 39 \\ &= 9 \end{aligned}$$

(b) Loops formula:

$$\begin{aligned} \mathrm{MO}_2 &= \sum_{i=1}^{k_2} \mathrm{MO}_{2,i} - 6\mathrm{bo}_2 \\ &= 39 - 6 \cdot 5 \\ &= 9 \end{aligned}$$

Exercise 4

Here is the kinematic representation of the robot NR-611 from NEC. One could recognize it as a Sarrus mechanism.



1. How many degrees of freedom do you think this robot has?
2. Calculate the mobility of the mechanism.
3. Make comments about your results.

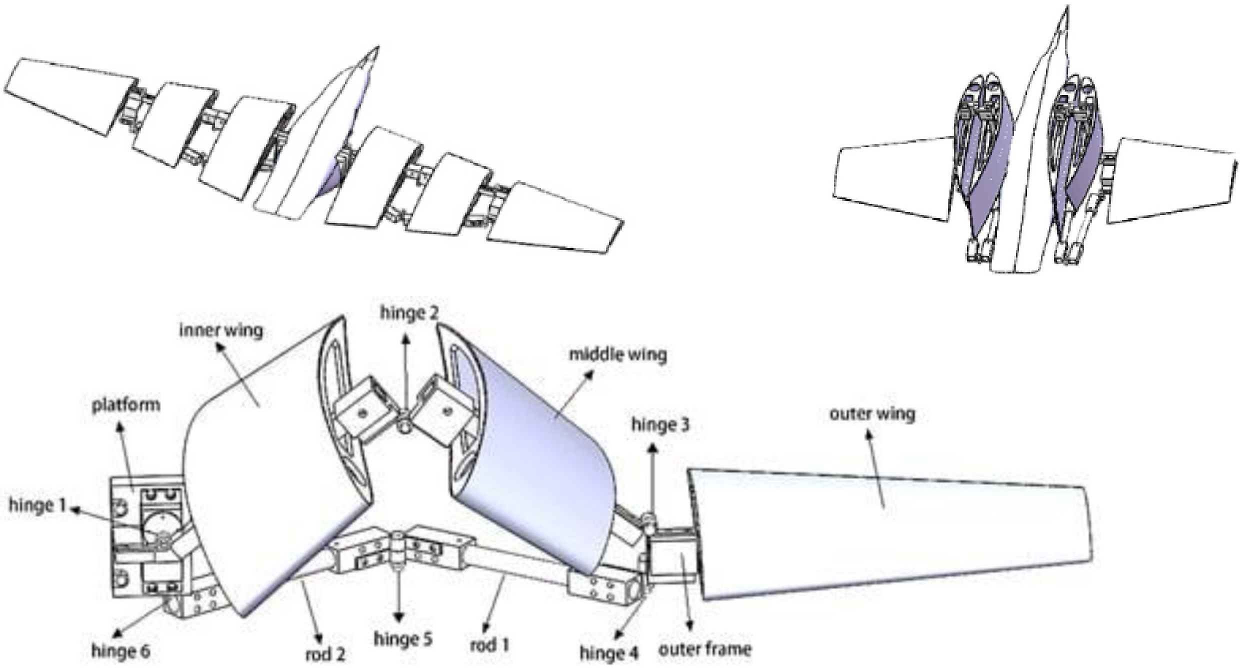
More examples with a Sarrus mechanism are given below to provide more intuition about the mechanism:

Example 1:

Fig. 1 Our mechanism, shown here attached to the skids on the bottom of an RC helicopter, uses a bilateral configuration of Sarrus-based linkages to passively grip cylindrical perches

[Source: Burroughs, M. L., Beauwen Freckleton, K., Abbott, J. J., and Minor, M. A. (August 18, 2015). "A Sarrus-Based Passive Mechanism for Rotorcraft Perching." ASME. J. Mechanisms Robotics. February 2016; 8(1): 011010.]

Example 2:



[Source: Yun Z, Feng Y, Tang X, Chen L. Analysis of Motion Characteristics of Bionic Morphing Wing Based on Sarrus Linkages. *Applied Sciences*. 2022; 12(12):6023.]

Also, you can check simulation of mechanism through the links:
https://www.youtube.com/watch?v=gfXWDGGip-0
https://www.youtube.com/shorts/pQBJcgJe6t0

Solution 4

1. The robot has 1 degree of freedom in translation. Each arm is constrained to move in a plane and the two planes of the two arms are not parallel. Therefore, the end effector, which is attached to the two arms, is constrained to move along a line. This line is the intersection of the two arms' planes.

2. We have n = 6 the number of parts (including base), k = 6 the number of joints and bo = 1 the number of closed loops. The MO_i are the mobilities associated with each joint, for i ranging from 1 to k, therefore:
$$\sum_{i=1}^{k} \mathrm{MO}_i = 6 \cdot 1 = 6.$$

   Grübler's formula:

$$\begin{aligned} \mathrm{MO} &= 6(n-k-1) + \sum_{i=1}^{k} \mathrm{MO}_i \\ &= 6(6-6-1) + 6 \\ &= 0 \end{aligned}$$

Loops formula:

$$\begin{aligned}\mathrm{MO} &= \sum_{i=1}^{k} \mathrm{MO}_i - 6\mathrm{bo} \\ &= 6 - 6 \\ &= 0\end{aligned}$$

2. There is an over-constraint (fr: hyperguidage): as you notice the calculated mobility is 0, but the robot has 1 degree of freedom. To avoid over-constraining this kinematic, one can:
   (a) Put a weakness in the gripper to achieve 1 additional MO between the two segments of the gripper or put a torsional weakness in one of the segments connecting the gripper.
   (b) Replace the current gripper by a gripper with an internal articulation, e.g. a hinge: